# CAPACITATED SCHEDULING OF MULTIPLE PRODUCTS ON A SINGLE PROCESSOR WITH SEQUENCE DEPENDENCIES

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Welch's, a major food manufacturer, must schedule its own and its co-packer's production lines. The scheduling process uses a 13-week rolling time horizon with weekly time buckets and schedules that are updated weekly. Each single stage finite capacity production line serves a dedicated group of products with sequence-dependent setup times and costs. In addition, safety stocks must be maintained to meet customer service requirements in the face of highly variable demand. Good low-cost schedules are difficult and time consuming to create "by hand," and optimal solutions are not possible using math programming methods except for very small problems.

In this article we describe a finite capacity planning system developed at Welch's that provides economical lot sizes by period and then sequences those lots within each period. The system is based on simple, intuitively appealing rules that do not require specialized knowledge of math programming or software. The production plans the system provides are not "optimal," but we will demonstrate that the costs are competitive with more sophisticated methods and provide substantial improvement over random schedules.

The system was developed by the Center for Process Manufacturing, a partnership of APICS and Penn State-Erie, with Welch's as a sponsoring member. The center is dedicated to helping decision makers in the process industry. Our goal, therefore, is to describe the scheduling system in sufficient detail so that others may adapt it to their own needs. Welch's implemented the scheduling procedure using an Excel spreadsheet with data downloaded from minicomputer databases. This approach gives production planners a powerful scheduling tool in the context of a familiar and convenient user interface, the computerized spreadsheet.

## PRODUCTION PLANNING AS A TWO-STEP PROCESS

Dilts and Ramsing [2] point out that the *simultaneous* solution of the lot-sizing and sequencing problem is "computationally prohibitive." That is somewhat of an understatement as each step, lot sizing and sequencing, is difficult to solve to optimality by itself for most business situations. Furthermore, for the lot-sizing problem with sequence *independent* setup times, you "cannot expect to find a fast algorithm to tell whether or not a feasible solution exists, let alone find an optimal solution." [5]. The sequencing problem for a given set of lot sizes is a variation of the traveling salesman problem (TSP), which requires a great deal of computing power to solve. When using the TSP approach for sequencing, we must "visit" each type of product scheduled for production within a time period, without backtracking, and at minimum cost.

The conclusion is that our scheduling problem must be addressed in two steps:

Step 1. Solve the lot-sizing problem using estimates for sequence-independent setup times and costs under conditions of finite capacity.

Step 2. Solve the sequencing problem using the true sequence-dependent setup costs.

According to Dilts and Ramsing [2], "this technique, while not guaranteeing an optimal solution, provides an acceptable solution to the problem."

Allen, et al. [1] show that the lot-sizing method used at Welch's created schedules with costs comparable to those obtained when mixed integer programming methods are used. We will now use data from one of Welch's product lines to demonstrate the sequencing step occurring after production quantities have been specified on a period-by-period basis. In a subsequent section, we will demonstrate the robustness of the method by generating and solving a variety of problems with the sequencing heuristic and then compare these solutions to those obtained using exact methods. It must be borne in mind that separate testing of the two stages, with cost as the performance measure, does not represent a test of the advantages of addressing lot sizing and sequencing simultaneously.

## DOING THE TWO-STEP

### Step 1: Lot Sizing Using Modified Dixon-Silver

Welch's uses the modified Dixon-Silver method (MODS), as reported by Allen, et al. [1], to carry out the lot-sizing process. MODS adapts the heuristic of Dixon and Silver [3] for zero setup times, accounting for sequence-independent setup times. The heuristic recognizes the limitations of finite capacity.

Clearly, when utilizing MODS to do lot sizing with sequence-dependent setup times, a critical factor will involve choice of a "representative" sequence-independent setup cost. [6] An example set of transition costs for a Welch's process is shown in table 1. Products are numbered 1 through 6, and transitions into or out of state 0 correspond to startups or shutdowns of the process.

Although table 1 shows transition costs, the issue when determining the production sequence, it should be pointed out that the transition time must be considered when specifying the size of the production lot. Because time spent changing between products represents lost capacity, production targets must be lowered to account for time the process spends in transition.

There are three common methods for accounting for the transition times when setting the production goals. The first uses the maximum transition time ("row maximum") a product might encounter as the representative setup time for each item. This approach guarantees sufficient machine time to make all transitions and meet the production quantities calculated with MODS. On the other hand, if we choose the second method, which uses the minimum transition time ("row minimum") as the representative setup time, it is likely that the production quantities will be too large, not leaving enough time to make the transitions. The row maximum approach is a conservative procedure, guaranteeing a feasible production schedule while introducing the possibility of having production capacity unaccounted for. The row minimum is an optimistic approach; production quantities from MODS can be too large, not leaving enough time to make all the transitions between products. The third approach is to provide the MODS routine with an average transition time. Based on our experience using Welch's data, we recommend using the row maximum value for representative setup times under conditions of high-capacity utilization. In situations of moderate to low levels of capacity utilization, we have had good luck using row averages as representative setup times.

### Step 2: Sequencing

To illustrate the sequencing process, we will consider a small example drawn from the Welch's data: a situation in which management has demand and inventory data for six types of items to be produced during four time periods. The output from Step 1, MODS, gives us a set of requirements by item and period; this is shown in table 2.

There are two common modes of operation at Welch's; therefore, two methods of sequencing are required:

Case 1: Each period begins and ends in the idle state, which we designate as state 0.

Case 2: Continuous production; the process seldom, if ever, returns to the idle state.

Case 1 is the most common situation encountered at Welch's and is the subject of this article. For Case 2

**TABLE 1: Setup Cost Matrix**

| | *To State*** | | | | | | |
|---|---|---|---|---|---|---|---|
| *From State* | *0* | *1* | *2* | *3* | *4* | *5* | *6* |
| 0 (Process Idle) | * | 150 | 110 | 120 | 100 | 130 | 100 |
| 1 (White Grape) | 150 | * | 90 | 100 | 100 | 150 | 100 |
| 2 (Fruit Harvest) | 110 | 500 | * | 500 | 500 | 500 | 500 |
| 3 (Grape Juice) | 100 | 200 | 70 | * | 100 | 200 | 100 |
| 4 (Grape Apple) | 100 | 200 | 70 | 100 | * | 500 | 100 |
| 5 (Harvest Blend) | 150 | 150 | 90 | 100 | 100 | * | 100 |
| 6 (Grape Raspberry) | 100 | 200 | 70 | 100 | 100 | 200 | * |

**State numbers are used to simplify the discussion.

**TABLE 2: Production Requirements by Item and Period**

| | *Period - t* | | | |
|---|---|---|---|---|
| *Item #, I* | *1* | *2* | *3* | *4* |
| 1 | 1 | | 1 | |
| 2 | | 1 | | 1 |
| 3 | | 1 | | 1 |
| 4 | 1 | 1 | | |
| 5 | | 1 | | 1 |
| 6 | 1 | | 1 | |

the sequencing rule must seek to find the least-cost path over the entire planning horizon. This means that testing simple sequencing rules against "optimal" sequences may not be possible for most problems encountered in practice.

We will frame our sequencing problem in terms of a traveling salesman problem by defining a *node* as the decision to produce a product in a time period. We can use arrows (directed arcs) between nodes to represent sequence paths. All production sequences in each period must begin and end in state 0. Figure 1, which shows the nodes that must be visited in each period, depicts an arbitrary production sequence. Our goal, however, is to find the lowest-cost set of arcs that visit each node once, and only once, within a time period. We will observe the following while constructing sequences.

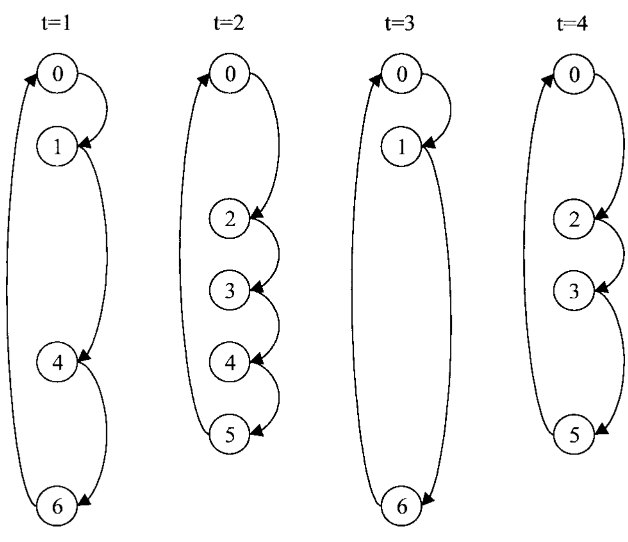


**FIGURE 1: Network representation of the sequencing problem**

1. All schedules must begin and end in state 0, the idle state.
2. All nodes (products scheduled to be produced within a time period) must be visited before transition to the next time period.
3. Table 1 will now serve as a setup cost matrix.

A possible sequence might be: 0-1-4-6-0-2-3-4-5-0-1-6-0-2-3-5-0, with a cost of $3,120. Using dynamic programming, we can easily show that the optimal solution is $1,720. The question is, can we find a simple procedure that will improve on the high cost of the arbitrary sequences shown above?

Gavett [4] examined three rules for solving the sequencing problem. We have tested his rules against Welch's production line data and found that nearest neighbor with variable origin (NNVO) consistently yielded the lowest cost sequences.

The NNVO algorithm uses nearest neighbor (NN) as a basic building block, and we describe it first.

NN Algorithm:
- Start at the beginning node.
  - Select among the reachable nodes with the lowest setup cost.
  - Rename this node the beginning node.
- Repeat until all nodes have been visited.

Using this rule we obtain a new sequence: 0-6-1-4-0-2-5-3-4-0-6-1-0-2-5-3-0, at a cost of $2,680. This sequence is an improvement over our earlier schedule, which cost $3,120, but is above the optimal value of $1,720.

NNVO Algorithm:
- Start at the beginning node.
  - For each node reachable from the beginning node,
    - Compute the cost to *complete the network* from that node using NN.
    - Add the cost of reaching that node from the beginning node.
    - Save the cost of that sequence.
  - Choose among the reachable nodes with the lowest cost.
  - Rename that node as the beginning node.
  - If only one node is reachable, rename that node the beginning node.
- Repeat until all nodes have been renamed.

This simple set of rules is much less myopic than NN because it examines the remainder of the network each time it selects a new beginning. Frequently the advantage offered by using all information contained in the cost matrix leads the NNVO algorithm to a lower cost sequence than the NN rule.

Next we demonstrate the use of NNVO sequencing for our example problem shown in figure 1.

### Sequencing with NNVO

Set t = 1
Beginning node: 0
Reachable nodes: 1, 4, 6

Node 1
NN sequence: 1-4-6-0
Cost = 300
Total cost = 150 + 300 = 450

Node 4
NN sequence: 4-6-1-0
Cost = 450
Total cost = 100 + 450 = 550

Node 6
NN sequence: 6-4-1-0
Cost = 450
Total cost = 100 + 450 = 550

Lowest cost is for Node 1
Beginning node: 1
Reachable nodes: 4, 6

Node 4
NN sequence: 4-6-0
Cost = 200
Total cost = 100 + 200 = 300

Node 6
NN sequence: 6-4-0
Cost = 200
Total cost = 100 + 200 = 300

Node 4 is the one with the lowest cost
Beginning node: 4
Reachable node: 6
Beginning node: 6
Stop

Final sequence: 0-1-4-6-0
Cost = 450

t = 2
Final sequence: 0-5-3-4-2-0
Cost = 510

t = 3
Final sequence: 0-1-6-0
Cost = 350

t = 4
Final sequence: 0-5-3-2-0
Cost = 410

The total cost of the sequences is $1,720, the optimal solution.

Although NNVO did find the optimal solution in our small example problem, there is no guarantee that it will always do so. To gain some insight into how the heuristic will perform in the range of operating characteristics likely to be encountered at Welch's, we will subject the heuristic to a much more rigorous testing procedure.

## TESTS OF NNVO

Our purpose now is to compare the costs of optimal sequences with the costs of sequences generated by NNVO. The procedure used by Gavett [4] will guide us here, but we will make adjustments appropriate for the situation at Welch's.

### Five Factors That May Influence NNVO

We have identified five factors that may influence the cost performance of the NNVO sequencing procedure and specified the performance ranges likely to be encountered.

1. The maximum number of items per period.
   This factor will be controlled at two levels: 8 and 20 items.
2. The mean number of items per period.
   This factor also will be set at two levels: 25% and 75% of the maximum allowable.
3. The per-period variability of the number of items.
   The number of items produced in each period is drawn from uniform distributions with the mean set at the level described in the prior two steps, and then the standard deviation is considered at two levels: $2/\sqrt{12}$ and $4/\sqrt{12}$.
4. Variability of setup costs.
   It is reasonable that the absolute level of the mean setup costs should not have an impact on the performance of the sequencing method. However, as Gavett [4] recognized, the variability might have an effect. We will hold the mean setup cost fixed at a value of 100 and select costs from a uniform distribution at two levels of standard deviation: $50/\sqrt{12}$ and $100/\sqrt{12}$.
5. Symmetry of setup costs.
   Gavett [4] also recognized that the degree of symmetry in the setup cost matrix may affect the quality of heuristic solutions. He suggested relaxing the requirement that the matrix be asymmetric. We will investigate the degree of symmetry of the setup cost matrix at two levels: symmetrical and asymmetrical. For the symmetrical case, a single transition cost, for off-diagonal terms, will be used

**TABLE 3: Experimental Design and Results**

| *Run #* | *Mean # Products Per Period* | *Variability of Products Per Period* | *Variability of Setup Costs* | *Symmetry of Setup Costs** | *Max. # of Products Per Period* | *% Error NNVO vs Optimal* | *%Error Random vs Optimal* |
|---|---|---|---|---|---|---|---|
| 1 | - | - | - | - | + | 1.02 | 27.64 |
| 2 | + | - | - | - | - | 6.63 | 31.28 |
| 3 | - | + | - | - | - | 0.00 | 24.29 |
| 4 | + | + | - | - | + | 6.49 | 71.72 |
| 5 | - | - | + | - | - | 0.00 | 22.58 |
| 6 | + | - | + | - | + | 39.36 | 392.44 |
| 7 | - | + | + | - | + | 6.39 | 129.71 |
| 8 | + | + | + | - | - | 14.80 | 171.40 |
| 9 | - | - | - | + | - | 0.34 | 1.24 |
| 10 | + | - | - | + | + | 1.59 | 22.97 |
| 11 | - | + | - | + | + | 0.01 | 8.05 |
| 12 | + | + | - | + | - | 1.05 | 10.34 |
| 13 | - | - | + | + | + | 0.00 | 14.74 |
| 14 | + | - | + | + | - | 1.40 | 20.07 |
| 15 | - | + | + | + | - | 0.00 | 8.26 |
| 16 | + | + | + | + | + | 3.70 | 58.00 |

*Low setting (-) represents the asymmetrical case.

for two matrix elements, Cij and Cji. For the asymmetrical case, each off-diagonal term will require a separate random selection, resulting in different values for Cij and Cji. To maximize the degree of asymmetry within the constraints of the stated mean and standard deviations, we developed the following process:

a. For *low* setup cost variability, define two uniform distributions:

LL = [50, 100]; LH = [100, 150].

b. Flip a coin to determine which distribution to use for Cij (of course, the computer will do the coin toss). Then select Cji from the other distribution.

c. Complete the same process for *high* setup cost variability using the two uniform distributions:

HL = [0, 100]; HH = [100, 200].

We are interested in differences in costs of sequences created using the NNVO procedure and those obtained from the optimal sequences. This led us to define our response (or dependent) variable as:

(Cost (NNVO) – Cost (Optimal)) / Cost (Optimal).

We used a 16-run one-half fraction of a $2^5$ factorial designed experiment. Results are given in the next section.

## RESULTS

Table 3 describes the levels for each of the five factors; a negative represents the low setting and a positive the high setting. The rightmost two columns list the percentage cost penalties for the sequences found with the NNVO heuristic and the randomly generated sequences.

The NNVO heuristic performed very well for 14 of the 16 test runs, producing optimal solutions in 5 cases. Run number 6 is particularly bothersome, though, with a cost penalty of $1,647 above the optimal value of $4,182. The largest errors appear to be associated with a high "density" of products per period and asymmetric setup costs. The maximum number of products per period and the level of variability (for both products per period and setup costs) appear to have little or no effect on the quality of NNVO sequences. This is borne out by regression analysis on the five factors and their interactions; only the main effects for mean number of products per period (density) and symmetry were significant, each with p-values of 7%.

## SUMMARY

Scheduling multi-item capacitated production lines with sequence dependent setup times and costs is a difficult problem, and optimal solutions are generally inaccessible. However, the simple two-step process we have described—lot-sizing using MODS [1] followed by sequencing using NNVO [4]—will provide good cost-based schedules. Nevertheless, we must recognize the potential for the occasional large cost penalty in Step 2. Unfortunately, we are not able to predict when these penalties will occur.

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